CHAPTER 7 – THE COST OF PRODUCTION

Key Concepts and Topics

- Measuring Cost: Which Costs Matter?
- Cost in the Short Run
- Cost in the Long Run
- Long-Run versus Short-Run Cost Curves
- Production with Two Outputs – Economies of Scope

Measuring Cost: Which Costs Matter?

- For a firm to *minimize costs*, we must clarify what is *meant* by cost and how to *measure* them
  - If a firm has to *rent* equipment or buildings, the *rent* they pay is a *cost*
  - What if a firm *owns* the equipment or building?
    - ♦ How are *costs* calculated here?

Economic Cost versus Accounting Cost

- Accounting Cost
  - A *retrospective* view of a firm's financing and operating costs
  - *Actual* expenses plus *depreciation* charges for capital equipment
- Economic Cost
  - A *forward-looking* view of a firm's operating costs
  - Cost to a firm of *utilizing economic resources* in production, including *opportunity cost*

Opportunity Cost

- Cost associated with opportunities that are *foregone* when a firm's resources are not put to their *highest-value* use (synonymous with *economic cost*)
- Example:
  - A firm *owns* its own building and pays *no rent* for office space
    - ♦ Does this mean the cost of office space is *zero*?
    - ♦ The building could have been *rented* instead
    - ♦ *Foregone rent* is the *opportunity cost* of using the building for production and should be included in *economic costs* of doing business

- A person starting his own business must take into account the *opportunity cost* of his *time*
  - Could have worked elsewhere making a *competitive salary*

Sunk Cost

- Expenditure that *has been made* and *cannot be recovered*
- Should not influence a firm's *future economic decisions*
- Example: Firm buys an equipment that cannot be converted to another use
  - Expenditure on the equipment is a *sunk cost*
  - Has no alternative use so cost cannot be recovered – opportunity cost is *zero*
  - Decision to buy the equipment might have been good or bad, but *now* does not matter

- Prospective Sunk Cost: An Example
  - A firm is considering moving its headquarters
  - The firm paid $500,000 for an option to buy a building
  - The cost of the building is $5 million or a total of $5.5 million
  - The firm finds another building for $5.25 million
  - *Which building should the firm buy?*
  - *The first building should be purchased*
  - The $500,000 is a *sunk cost* and should *not be considered* in the decision to buy
  - What should be considered is
    - Spending an additional $5,250,000 or $*5,000,000*

Fixed and Variable Costs

- Fixed Cost (*FC*)
  - Does not *vary* with the *level of output*
  - Costs paid by the firm to be *in business* (e.g., *plant maintenance, salaries, rents*, etc.)
  - Can be eliminated only when the firm *goes out of business*

- Variable Cost (*VC*)
  - Cost that *varies* as *output varies* (e.g., *raw materials, wages*, etc.)

- Total Cost (TC) of production equals the fixed cost (the cost of the *fixed inputs*) plus the variable cost (the cost of the *variable inputs*), i.e., $TC = FC + VC$

- Which costs are variable and which are fixed depends on the *time horizon*
  - *Short time horizon*: most costs are *fixed*
  - *Long time horizon*: many costs become *variable*

- Important to understand the *characteristics* of production costs and be able to identify which costs are *fixed*, *variable* and *sunk*, because a firm's profitability relies on its *cost structure*. The *relative sizes* of these cost components vary significantly across industries:
  - Personal Computers
    - Most costs are *variable*
    - Largest component: *labor*
  - Software
    - Most costs are *sunk*
    - Initial cost of *developing the software*

Marginal and Average Cost

- Marginal Cost (*MC*) (or *incremental* cost)
  - The cost of expanding output by *one unit*

$$MC = \frac{\Delta VC}{\Delta q} = \frac{\Delta TC}{\Delta q}$$

  - Can be measured by the *slope* of the *tangent* to the *VC* curve at the corresponding point

- Average Total Cost (*ATC*) (*AC* or average *economic* cost)
  - Cost per *unit of output*

$$ATC = \frac{TC}{q} = AFC + AVC = \frac{TFC}{q} + \frac{TVC}{q}$$

- Average Fixed Cost (*AFC*)
  - Fixed cost divided by the *output level*: $AFC = \frac{FC}{q}$
  - *Declines* as output *increases* because *FC does not change* with *q*

- Average Variable Cost (*AVC*)
  - Variable cost divided by the *output level*: $AVC = \frac{VC}{q}$
  - Can be measured by the *slope* of the line drawn from the *origin* to the corresponding point on the *VC* curve

Cost in the Short Run

- In the short run, most costs are *fixed*
- A Firm's short run costs

q	FC	VC	TC FC+VC	MC ΔTC/Δq ΔVC/Δq	AFC FC/q	AVC VC/q	ATC TC/q AFC+AVC TFC/q+TVC/q
0	50	0		–	–	–	–
1	50	50					
2	50	78					
3	50	98					
4	50	112					
5	50	130					
6	50	150					
7	50	175					
8	50	204					
9	50	242					
10	50	300					
11	50	385					

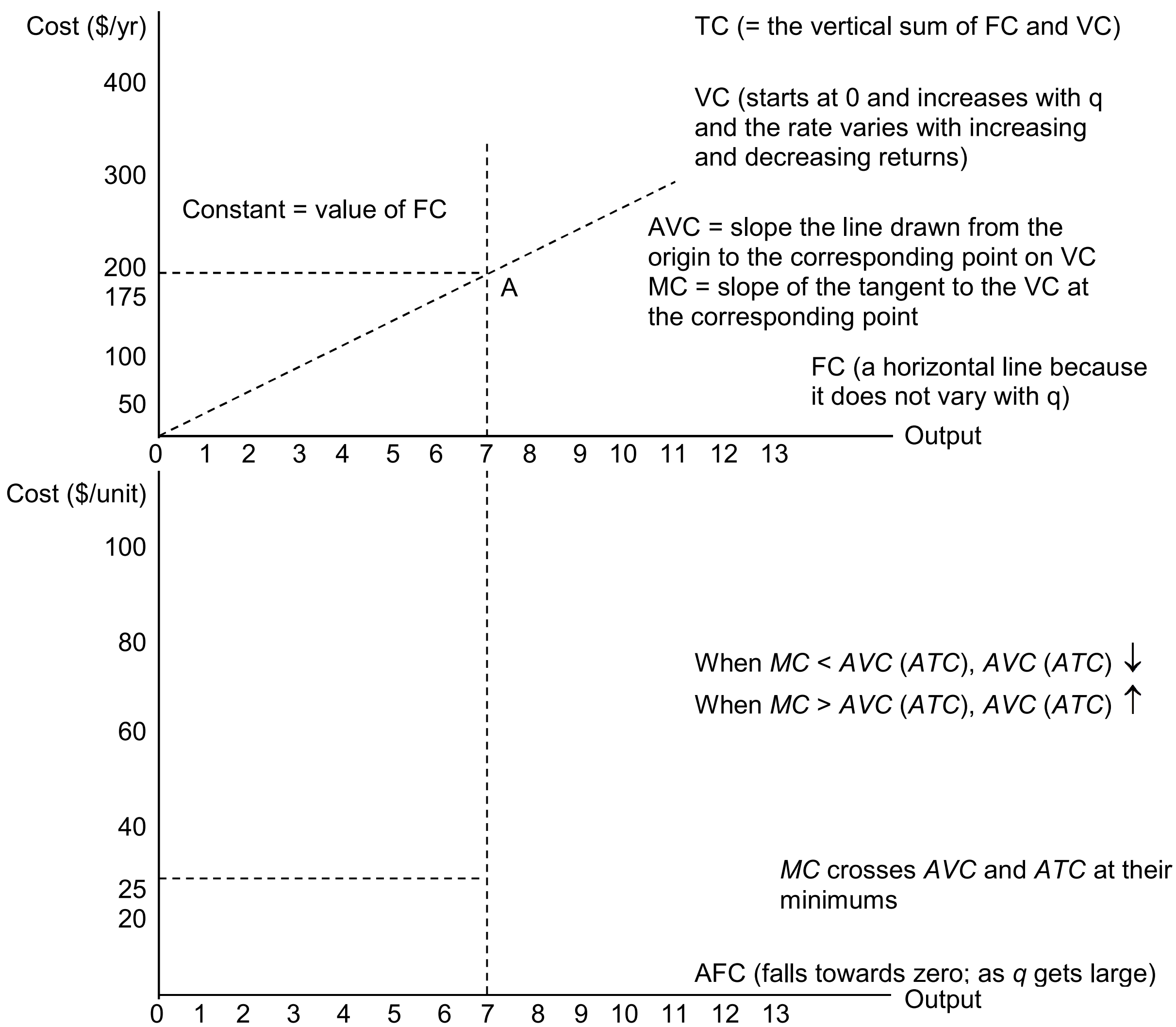
  - *MC decreases* initially with *increasing* returns
  - *0* through *4* units of output
  - *MC increases* with *decreasing* returns
  - *5* through *11* units of output

- Determinants of Short-run Costs
  - The rate at which variable and total costs increase depends on the nature of the *production process*
  - The extent of *diminishing marginal returns* of the production factors involved

- Diminishing marginal returns
  - The marginal product of input *falls* as *more* and *more* of that input is used
  - Marginal cost *increases* as output *rises*
  - Example: Diminishing marginal returns to labor
    - If MP_L *decreases* significantly as *more* labor is hired
      - Costs of production *increase* rapidly
      - Greater and greater *expenditures* must be made to produce *more* output
    - If MP_L *decreases* only slightly as *increase* labor
      - Costs will not *rise* very fast when output is *increased*

- The Relationship between Production and Cost
  - Labor cost is a *fixed* wage rate, w
  - Marginal cost is the change in *variable* cost for *one-unit* change in *output*, $MC = \Delta VC / \Delta q$
  - Change in variable cost is the *per unit* cost of extra labor times the amount of *extra labor* required to produce the *extra* unit of output: $\Delta VC = w \Delta L$
  - Marginal product of labor is the change in *output* resulting from *one-unit* change in labor input, $MP_L = \dfrac{\Delta q}{\Delta L} \Rightarrow \dfrac{\Delta L}{\Delta q} = \dfrac{1}{MP_L}$
  - We can conclude that *marginal cost* is the *price* of the input (labor) divided by its *marginal product*, $MC = \dfrac{\Delta VC}{\Delta q} = \dfrac{w \Delta L}{\Delta q} = \dfrac{w}{MP_L}$ and *a low marginal product (MP) leads to a high marginal cost (MC), and vice versa*

- The Shapes of the Cost Curves



Source: Pindyck and Rubinfeld (2009), *Microeconomics*, 7th Ed., Pearson Prentice Hall, Chapter 7.

- The Average-Marginal Relationship
  - When $MC < AVC$ (ATC), AVC (ATC) falls until $MC = AVC$ (ATC)
  - When $MC = AVC$ (ATC), AVC (ATC) is at its minimum
  - When $MC > AVC$ (ATC), AVC (ATC) rises

Cost in the Long Run

- In the long run, a firm can change *all* of its inputs
- In making *cost minimizing* choices, a firm must look at the *cost* of using capital and labor in *production* decisions

- Capital is either *rented/leased* or *purchased*
  - We will consider capital *rented* as if it were *purchased*

- User Cost of Capital
  - The annual cost of *owning* and *using* the capital asset
  - Sum of the *economic depreciation* (the *purchased price* of the capital asset divided by the *life* of the asset) and the *interest* (the *financial* return) that could have been *earned* had the money been *invested* elsewhere
  - *Forgone interest* is an *opportunity cost* and must be included in the cost
  - As the capital asset *depreciates* over time, the *opportunity cost* of the financial capital that is invested in it *decreases*
  - User Cost of Capital = *Economic Depreciation* + (*Interest Rate*)*(*Value of Capital*)

- Example: Air Canada is considering purchasing an airplane for $150 million
  - Plane lasts for 30 years
  - $5 million per year – *economic depreciation* for the plane
  - Air Canada needs to compare its revenues and costs on an annual basis
  - If the firm had not purchased the plane, it would have earned *interest*, say 10 percent, on the $150 million
    - User Cost of Capital = *$5 million* + (*.10*)(*$150 mil – depreciation*)
      - Year 1 = *$5 million* + (*.10*)(*$150 million*) = *$20 million*
      - Year 10 = *$5 million* +(*.10*)(*$100 million*) = *$15 million*

- User cost can also be described as a *rate per dollar of capital*, *r*
  - *r* = *Depreciation* rate + *Interest* rate
    - In our example, depreciation rate was *3.33*% and *interest* was *10*%
    - *r* = *3.33*% + *10*% = *13.33*%

- Cost Minimizing Input Choice
  - The *optimal* amount of inputs used to produce a *given* output at the *minimum* cost depends on the *prices* of the inputs
  - Assuming two inputs: Labor (L) and capital (K)
    - Price of labor: wage rate (w)
    - The price of capital (r)
      - r = *depreciation* rate + *interest* rate
      - Or, *rental rate* if not purchased
      - These are *equal* in a competitive capital market

- Rental rate of capital
  - The *cost* per year of *renting* one unit of capital (e.g., *the rental rate of office space in an office building*)
  - In a competitive market, owner of the rental capital expects to earn a *competitive* return (i.e., the rate of return that he could have earned by *investing* his money elsewhere, plus an amount to compensate for the *depreciation* of the capital asset), which is equivalent to the *user cost* of capital
  - It is reasonable to assume that the rental rate of capital is *equal* to the *user cost* of capital, r

- The Isocost Line
  - Shows all *possible combinations* of inputs (L and K) that can be purchased for a *given total cost*
  - Total cost of production (C) is the *sum* of labor cost, wL, and capital cost, rK

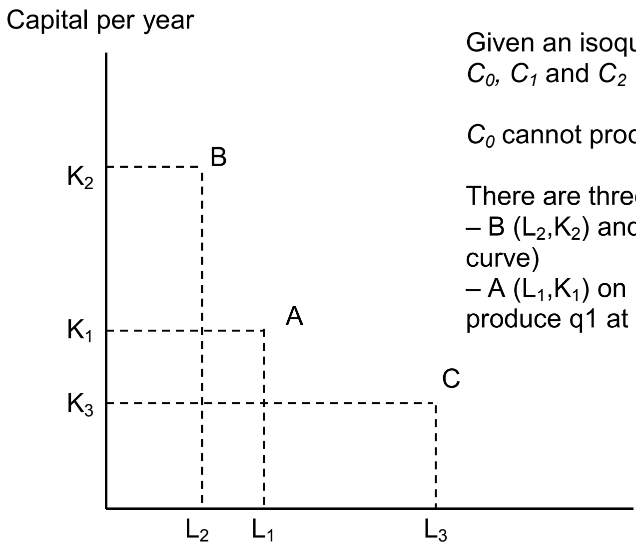
$$C = wL + rK$$

  - Rewriting C as an equation for a *straight line*

$$K = \frac{C}{r} - \frac{w}{r}L$$

  - Slope of the isocost line = $\frac{\Delta K}{\Delta L} = -\frac{w}{r}$, the ratio of the *wage rate* to the *rental cost* of capital, shows the rate at which *capital* can be substituted for *labor* with *no change* in cost
    - Changes on *expenditures* do not alter the *slope* of the isocost line because the *input prices* remain unchanged
    - If the *price* of one input (labor) were to *increase*, the *slope* of the isocost line would *increase* and become *steeper*, or vice versa

- Choosing Inputs
  - The *cost-minimizing* choice of inputs, *L* and *K*, to produce a *given* output *q* is at the point of *tangency* of the *isoquant* and the *lowest isocost line* that allows output *q*
  - At that point, the *slope* of the *isoquant* is *equal* to the *slope* of the *isocost line*
  - Producing a given output at minimum cost

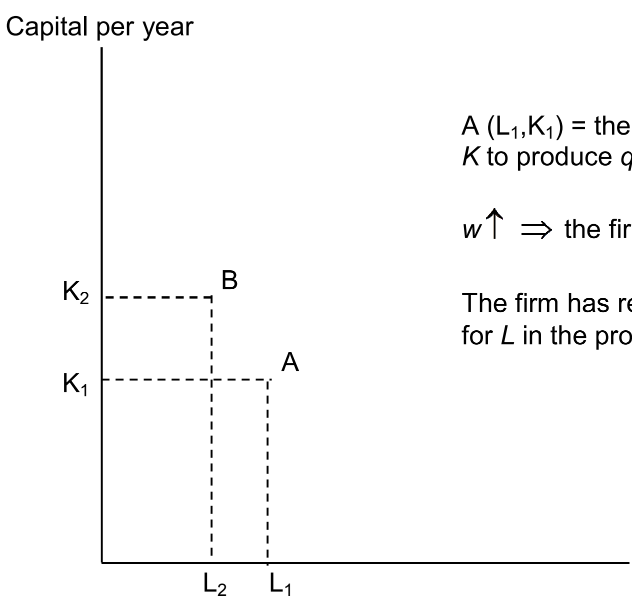


Given an isoquants q_1 and three isocost lines C_0, C_1 and C_2

C_0 cannot produce *q1*

There are three possible choices to produce q_1
– B (L_2,K_2) and C (L_3,K_3) on C_2 (the highest cost curve)
– A (L_1,K_1) on C_1, tangent to isoquant q1, produce q1 at the cheapest cost

- Input Substitution When an Input Price Changes
  - The *slope* of the *isocost line* changes when the *price* of labor changes
  - The *optimal* choice of *L* and *K* to produce output q_1 would change to a new *tangency* of the *isoquant* and another *isocost line*, where *more K* and *less L* are used (i.e., substituting the *relatively higher* price of labor with the *relatively cheaper* price of capital)



A (L_1,K_1) = the initial cost-minimizing choice of *L* and *K* to produce q_1

$w\uparrow \Rightarrow$ the firm chooses B (L_2,K_2) to produce q_1

The firm has responded to the $\uparrow$ w by substituting *K* for *L* in the production process

- The Relationship between the Isocost Line and the Firm's Production Process
  – $MRTS_{LK} = -\frac{\Delta K}{\Delta L} = \frac{MP_L}{MP_K}$ and the slope of the isocost line $= \frac{\Delta K}{\Delta L} = -\frac{w}{r}$
  – Cost *minimization* occurs when the *slope* of isoquants = the *slope* of the isocost line (i.e., at their *point of tangency*)

$$\frac{MP_L}{MP_K} = \frac{w}{r}$$

$$\Rightarrow \frac{MP_L}{w} = \frac{MP_K}{r}$$

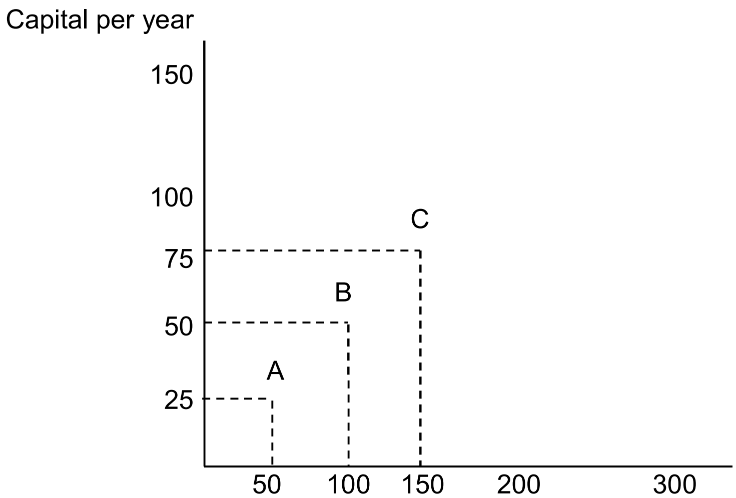
  – This *cost-minimizing* condition implies that, to produce a *given* output, a firm should choose the quantities of inputs so that *the last dollar spent on any input yields the same amount of additional output*
  – Example: If w = \$10, r = \$2, and $MP_L = MP_K = 20$, which input would the producer use more of?
    - ♦ Cost minimization occurs when

$$\frac{MP_L}{w} = \frac{MP_K}{r}$$

    - ♦ Given $\frac{MP_L}{w} = \frac{20}{10} = 2 < \frac{MP_K}{r} = \frac{20}{2} = 10$, the producer should use *more K* and *less L* because *K* is *5 times more productive than L*
    - ♦ The *increase* use of *K lowers* MP_K and the *decrease* use of *L raises* MP_L
    - ♦ The producer should continue to *substitute K* for *L* until it reaches the *cost-minimizing* condition: $\frac{MP_L}{w} = \frac{MP_K}{r}$

- Cost Minimization with Varying Output Levels
  – For each level of output, the cost-minimizing input quantities can be determined at the *point of tangency* between the *lowest isocost line* and the *isoquants* for that output level
  – A firm's *expansion path* shows the cost-minimization choices of *L* and *K* at each level of output
    - ♦ Drawn by passing through the *points of tangency* between the firm's *isocost lines* and *isoquants*
    - ♦ *Slopes upward* as long as the *increase* use of inputs *increases* output
    - ♦ Slope equals $\Delta K/\Delta L$

– A Firm's Expansion Path

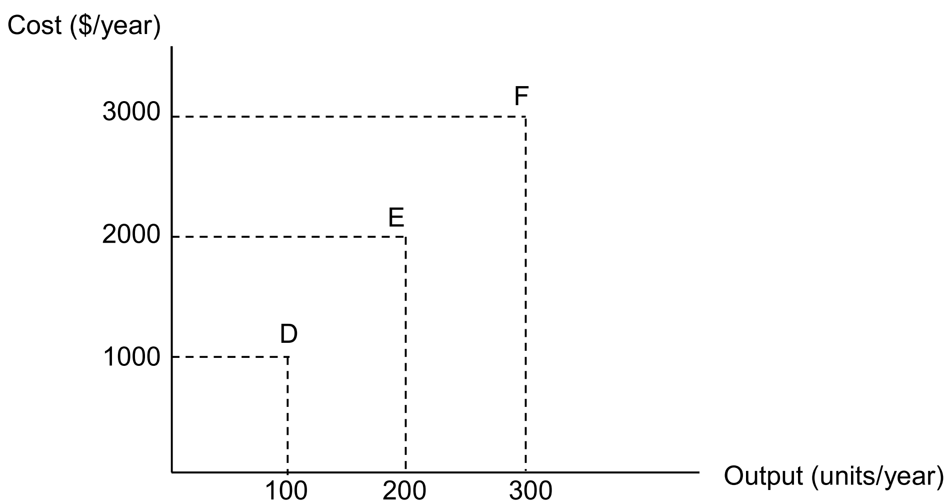


The expansion path illustrates the lowest-cost combinations of *L* and *K* that can be used to produce each level of output in the long run (i.e., when both inputs to production can be varied)

A, B and C show the least cost combinations of *L* and *K* to produce 100, 200 and 300 units of output, respectively.

- Expansion Path and Long-run Costs
  - The long-run total cost curve shows the *output-cost* combinations based on the information provided by the *expansion path*
    - Illustrates the *least* long-run cost of producing each level of output
    - *Slopes upward*

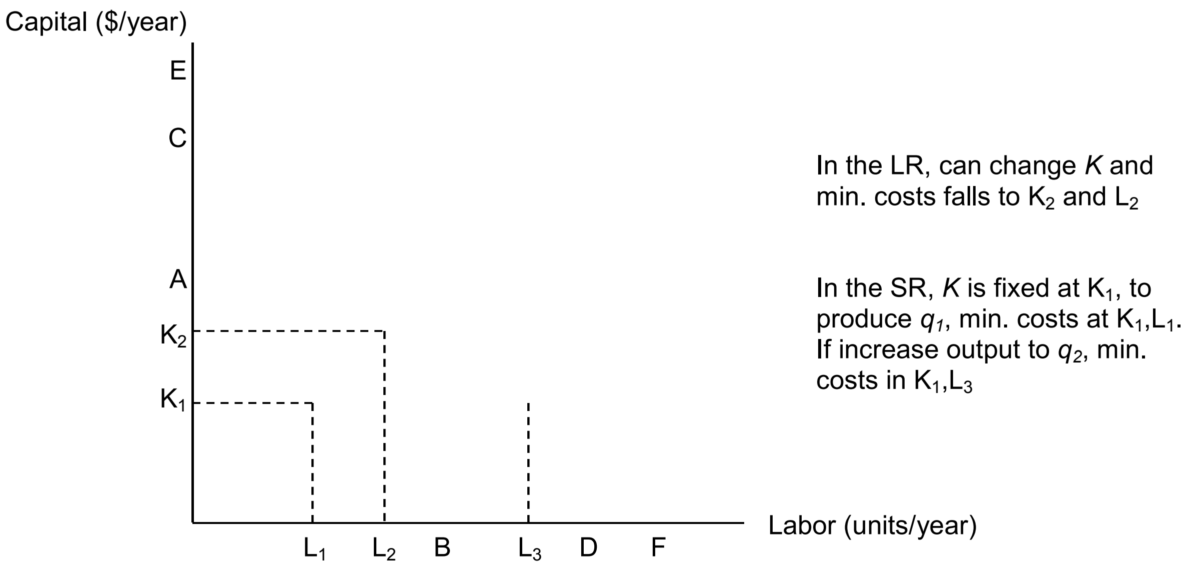
– A Firm's Long-Run Total Cost Curve



Source: Pindyck and Rubinfeld (2009), *Microeconomics*, 7th Ed., Pearson Prentice Hall, Chapter 7.

Long-Run Versus Short-Run Cost Curves

- The Inflexibility of Short-Run Production
  - In the short run, cost of production may not be *minimized* because of *inflexibility* in the use of *capital* input
    - The short-run expansion path first *rises* from the origin to the *fixed capital* input and then becomes a *horizontal* line because *higher output* level can be achieved only by *increasing* labor input
  - In the long run, the firm can vary *all* inputs, which allows the firm to expand production at the *least cost* by *substituting* the *relatively cheaper* input for *more expensive* input
    - The long-run expansion path is a *straight line* from the origin passing through *points of tangency* between the firm's *isocost lines* and *isoquants*

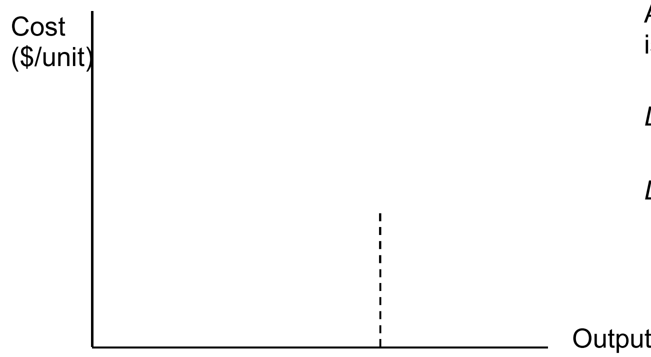


- Long-Run Average Cost
  - The relationship between the *scale* of the firm's operation and the *inputs* that are required to *minimize* costs determines the *shape* of the LR *AC* and *MC* curves
    - *Constant* Returns to Scale
      - *AC* are *constant* at all levels of output.
    - *Increasing* Returns to Scale
      - *AC decreases* with output
    - *Decreasing* Returns to Scale
      - *AC increases* with output

- Long-Run Average Cost Curve (*LAC*)
  - Relates average cost of production to *output* when *all* inputs, including capital, are *variable*
    - U-shaped because of *increasing* and *decreasing* returns to scale
    - At its *minimum* when *intersects* with the long-run *marginal cost* curve

- Short-Run Average Cost Curve (*SAC*)
  - Relates average cost of production to *output* when level of capital is *fixed*
    - U-shaped because of *diminishing returns* to input

- Long-Run Marginal Cost Curve (*LMC*)
  - Measures the *change* in long-run total cost as *output* is increased *incrementally* by *one unit*



At A, *LMC* crosses *LAC*, *LAC* is at its minimum

LAC ↓ when *LAC* > *LMC*

LAC ↑ when *LAC* < *LMC*

- Long-Run Average and Marginal Cost
  - When *LMC* < *LAC*, *LAC* is *falling*
  - When *LMC* > *LAC*, *LAC* is *rising*
  - When *LMC* = *LAC*, *LAC* is at its *minimum*
  - When *LAC* is *constant*, *LMC* and *LAC* are *equal*

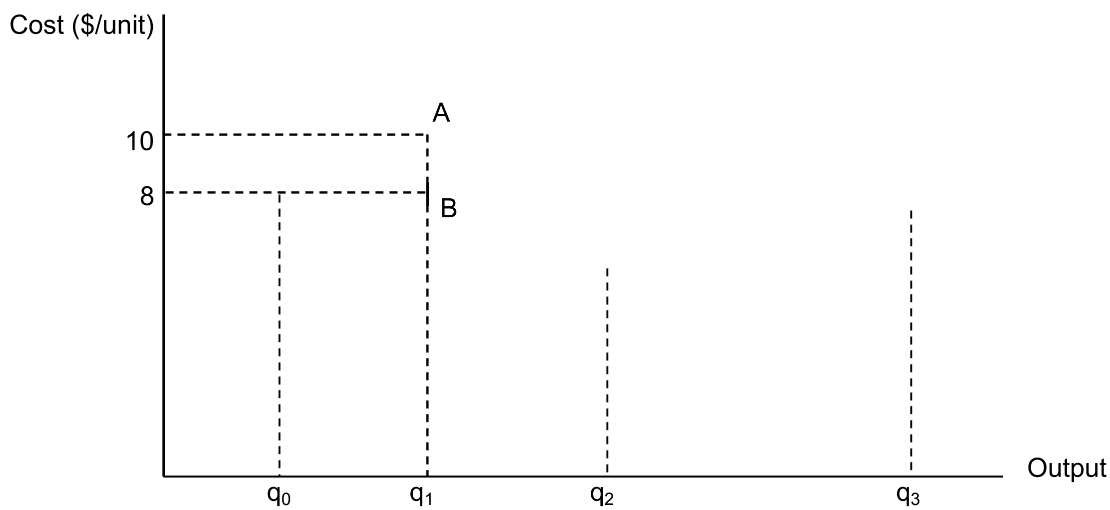
- Economies and Diseconomies of Scale
  - Economies of Scale
    - *Increase* in output is *greater* than the *increase* in inputs
  - Diseconomies of Scale
    - *Increase* in output is *less* than the *increase* in inputs
  - U-shaped *LAC* shows *economies* of scale for *relatively low* output levels and *diseconomies* of scale for *higher* levels
  - Measured in terms of *cost-output* elasticity, E_C
    - E_C is the percentage change in the *cost of production* resulting from a one-percent increase in *output*

$$E_C = \frac{\Delta C / C}{\Delta q / q} = \frac{\Delta C / \Delta q}{C / q} = \frac{MC}{AC}$$

- $E_C = 1$ (i.e., $MC = AC$)
  - Costs *increase proportionally* with output
  - *Neither economies nor diseconomies* of scale
- $E_C < 1$ (i.e., $MC < AC$)
  - *Economies* of scale
- $E_C > 1$ (i.e., $MC > AC$)
  - *Diseconomies* of scale

– A special case of economies of scale: *increasing returns to scale*
  - Returns to scale – input proportions are *fixed*
  - Economies of scale – input proportions are *variable*

- The Relationship between Short-Run and Long-Run Cost
  – Illustrate through the case that a firm is uncertain about the future demand for its product and is considering three alternative plant sizes
  – Use *short* and *long-run* cost to determine the *optimal* plant size
  – This decision is important because *once built*, the firm may not be able to change *plant size for a while*

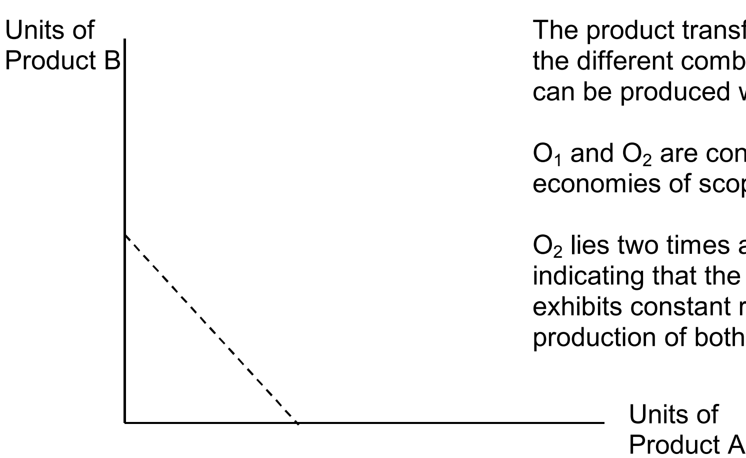


– The *optimal* plant size will depend on the *anticipated* output
  - If expect to produce q_0, then should build the *smallest* plant: $AC = \$8$
  - If then produce q_1, AC is still \$*8* with the *smallest* plant, it would be \$10 with the *middle* plant
  - If expect to produce q_2, the *middle* plant costs least
  - If expect to produce q_3, the *largest* plant is best

- What is the firms' long-run cost curve?
  - Only three possible plant sizes
    - In the long run, the firm can change the *plant size* and choose the plant that *minimizes* the *AC* of production
    - The *LAC* is the *crosshatched* portions of the *SAC*s, which represents the *minimum* cost for any level of output – the *envelope* of the *SAC*s
  - All possible plant sizes
    - The *LAC* is *U-shaped*, which reflects the presence of *economies of scale* initially and *diseconomies* at higher output levels
    - The *LAC* lies *below* the *SAC*s
    - The *minimums* of the *SAC*s do not *lie on* the *LAC* because larger plants can benefit from *economies of scale* to produce at a *lower AC*, which does not exist in *smaller* plant
- The *LMC* is not the *envelope* of the *SMC*s, because *LMC* works for all *possible* plant sizes while *SMC* works for a *particular* plant. Every point on the *LMC* is the *SMC* associated with the most *cost-efficient* plant

Production with Two Outputs – Economies of Scope

- Product Transformation Curve (*PTC*)
  - Shows the *various combinations* of two different outputs (products) that can be produced with a *given set of inputs*
  - Slopes *downward* because *getting more* of one output would require giving up some of the other output
    - A *straight-line PTC* shows *no gains or losses* from the joint production
    - A *concave (bowed outward) PTC* shows joint production by a single firm has *advantages* over the products being produced separately by individual firms using the *same resources*



The product transformation curve describes the different combinations of two outputs that can be produced with a fixed amount of inputs.

O_1 and O_2 are concave because there are economies of scope in production

O_2 lies two times as far from the origin as O_1 indicating that the firm's production process exhibits constant returns to scale in the production of both products

- Economies and Diseconomies of Scope
  - Economies of scope
    - The joint output of a single firm is *greater* than the output that could be produced by two different firms when each produces a single product
  - Diseconomies of scope
    - The joint output of a single firm is *less* than the output that could be produced by two separate firms when each produces a single product
  - There is no *direct relationship* between economies of scope and economies of scale.
    - May experience economies of scope *and* diseconomies of scale
    - May have economies of scale *and not have* economies of scope
- The degree of economies of scope (*SC*)
  - Measured by the percentage of *savings in cost* resulting from producing two or more products jointly

$$SC = \frac{C(q_1) + C(q_2) - C(q_1, q_2)}{C(q_1, q_2)}$$

  where $C(q_1)$ and $C(q_2)$ are the separate costs of producing q_1 and q_2, respectively, and $C(q_1,q_2)$ is the joint cost of producing both products

    - When $SC > 0$, there are economies of scope
    - When $SC < 0$, there are *diseconomies* of scope
    - When $SC = 0$, there is *neither economies nor diseconomies* of scope
    - The *greater* the value of *SC*, the *greater* the economies of scope

Quick Quiz

1. Please explain whether the following statements are true or false.
   a. If the owner of a business pays himself no salary, then the accounting cost is zero, but the economic cost is positive.
   b. A firm that has positive accounting profit does not necessarily have positive economic profit.
   c. If a firm hires a currently unemployed worker, the opportunity cost of utilizing the worker's services is zero.

2. If a firm enjoys economies of scale up to a certain output level, and then cost increases proportionately with output, what can you say about the shape of the long-run average cost curve?

3. Assume the marginal cost of production is increasing. Can you determine whether the average variable cost is increasing or decreasing? Explain.

4. Is the firm's expansion path always a straight line?

5. A firm has a fixed production cost of $5,000 and a constant marginal cost of production of $500 per unit produced.
   a. What is the firm's total cost function? Average cost?
   b. If the firm wanted to minimize the average total cost, would it choose to be very large or very small? Explain.

6. You manage a plant that mass produces engines by teams of workers using assembly machines. The technology is summarized by the production function.

$$q = 5KL$$

where q is the number of engines per week, K is the number of assembly machines, and L is the number of labor teams. Each assembly machine rents for r = $10,000 per week and each team costs w = $5,000 per week. Engine costs are given by the cost of labor teams and machines, plus $2,000 per engine for raw materials. Your plant has a fixed installation of 5 assembly machines as part of its design.
   a. What is the cost function for your plant – namely, how much would it cost to produce q engines? What are average and marginal costs for producing q engines? How do average costs vary with output?
   b. How many teams are required to produce 1,000 engines? What is the average cost per engine?

Source: Pindyck and Rubinfeld (2009), *Microeconomics*, 7th Ed., Pearson Prentice Hall, Chapter 7.

c. You are asked to make recommendations for the design of a new production facility. What capital/labor (K/L) ratio should the new plant accommodate if it wants to minimize the total cost of producing any level of output q?
d. How many labor teams and assembly machines are required to produce 1,000 engines at the minimum cost? What is the average cost per engine?
e. What is the cost function for producing q engines at the minimum cost?